

**AG313 Summary:**

**Treasury Management & Derivatives**

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AG313: Treasury Management & Derivatives

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**AG313 Course Summary**

**Derivatives**

**1: Options**

**1.1: Option vs. Forward Contracts:**

* Forward: Obligation to buy/sell in the future at rate
  + Future ER Safety
* Option: Right to buy/sell in future at rate
  + No Future ER Safety (may be better/worse)

**1.2: Spot vs. Future/Forward Prices**:

* Spot Price: immediate delivery price (***S0, ST***)
* Future/Forward Price: future delivery price (lock in price today) (***F0, FT***)
  + *FT* (<) *ST*: Forward is Spot grossed up at *r*, Spot exp. to be (>) *r* growth

**1.3: Short vs. Long Positions:**

* Short: Sells shares **now** (***S0*** = Spot), buy later (***ST*** = Delivery)
  + Exp. fall in **future** to buy
  + Futures price up: loss
* Long: Buy shares **now**
  + Exp. rise in **future** to sell
  + Futures price up: gain

**1.4: Call vs. Put Option Types:**

* **At-The-Money**:
* Call:Agreement to buy at specified time and Strike Price
  + Profit **(In-The-Money)** When
* Put: Agreement to sell at specified time and Strike Price
  + Profit (**In-The-Money**) When
* European Option: exercised only on expiration
* American Option: exercised any time up-to expiration

**1.5: Exchange vs. Over-the-Counter**:

* Over-the-Counter: $600tn; high credit risk – high prices
  + Trades Forward Contracts
* Exchange: $60tn; more standardised and regulated
  + Trades Futures Contracts

**2: Futures Market**

* Regulated by ‘Commodities Futures Trading Commission (CFTC)’
* **Clearing** **House**: always used in Futures Market to ensure payment made
* **Central Clearing Parties (CCPs)**: similar job to CH
* **Haircut**: difference between Market Value and Collateral usage of an asset
* **Bilateral Clearing**: group agree terms to trade w/ each other to minimise risk
* **Limit Order**: trader identifies worst at which trade can take place
* *Note that Futures on stock are cosh settled as opposed to w/ the underlying asset*

**2.1: Forward vs. Future**:

* Futurestrade on Exchange Market – standardised
* Futures last shorter time than Forward Contracts
* Futures don’t usually have final cash settlements
* Short: loss when futures rises – must buy at higher price to replace
* Long: gain when futures rises – made profit as share price rises

**2.2: Margin ‘Curtain Call’ Call**:

* *Broker’s demand that investor adds funds to retain minimum value of fund -* ***daily***
* Options up to 9-months must be bought in full, post-9: margin can be taken
* The **seller** posts margin, not the buyer as they make the payment for the option
* **Margin** **Accounts** are adjusted for gain/loss daily
* Reduce Systematic Risk 🡪 Ensure Funds Available 🡪 Reduce Risk of Back-Out
* Margin Call when loss over: (Initial Margin (-) Maintenance Margin)
  1. If Short: ea. $1 rise in price is a $1 per unit loss – find (=) to above
  2. Add the per unit rise to the per unit price
  3. If Long: ea. $1 rise in price is a $1 per unit gain – find (=) to above
  4. Add the per unit rise to the per unit price

**2.3: Corn** **Futures** **Contract**:

* Initiated by: party w/ Short position – ‘Notice of Intention to Deliver’
* Exchange goes through procedure of choosing party to take Long position

**2.4: Hedging vs. Speculating:**

* Hedging: e.g. expect volatility perhaps price rise to take Futures contract to lock in a price now
* Speculating: e.g. act upon expectation perhaps where they expect a fall in price, they’d take a Short position and buy back for profit
* *Hedgers hold Long, Speculators hold Short: (FT > ST)*

**3: Forward & Futures Prices**

* *Futures Price quoted as no. of US$/unit of foreign currency*
* *Lenders cannot issue instructions*
* **Investment Asset**: traded but not usually physically usable or tangible
* **Consumption Asset**: traded and usable for consumption (e.g. Copper)
  + Upper limit but no lower limit
* **Convenience Yield**: 0/(+), measures benefit of owning rather than for./fut.
  + *Having* ***real*** *value vs.* ***locked in F*** *value*
  + Investment: (=)0
  + Consumption: (+)
  + **Increase**: *F* as % of *S* decreases; more convenient to own
  + **Decrease**: *F* as % of *S* increases; more convenient to *F*
* **Div. Yield**: Div.’s as a % of Stock Price at *t* of Div. payment
* **Contango**: *FT* (>) *ST* abnormal

**3.1: Shorting w/ Dividends:**

* (1) *S0* Sell Now *ST* Buy Later (*S0* – *ST* Gain/Share) 🡪 (2) Pays Div. (*S0* – *ST* – Div)

**3.2: Spot to Forward Price:**

**4: Hedging Strategies w/ Futures**

* *Futures delivery month should be as close as possible to purchase of asset*
* *“Tailing the Hedge”*: Corrects for daily settlement
* Hedging Futures leads to predictability

**5: Options Market**

* **Recall**: *“The option, but not obligation, to buy (call) or sell (put)”*
* **Recall**:
  + Short: sell now (exp. to buy later lower)
  + Long: buy now (exp. to sell later higher)
* **Recall**: differences in *S,* *E(S),* and *F* mean Short Call, Long Calls, Short Put, Long Put are all different
* **Option Class**: *“All Calls or Puts on a stock”*
* **Option Series**: *“All options on a certain stock* ***type****”*
* **LEAPS**: Long-Term Equity Anticipation Securities w/ long maturities
* **Stock-Split**:
  + E.g. N = 100, K = 20, 2 for 1 Split
  + Ans. N = 2(100) = 200, K = ½(20) = 10
* **Stock-Div**:
  + E.g. N = 100, K = 20, 25% Div.
  + Ans. N = 1.25(100) = 125, K = 4/5(20) = 16
* **Cash-Div**:
  + No Effect
* Option Value (=) Time Value (+) Intrinsic Value
  + At-The-Money Time Value (=) 0 so Option Value (=) Intrinsic Value
  + Call:
  + Put:

**6: Option Pricing**

**1: Binomial Option Tree – European Put**

**Step 1**

**Step 1**

**Step 3**

**2: Converting to American Put**

**7: Stock Options**

* Stock Price (↑): Call (↑), Put (↓)
* Strike Price (↑): Call (↓), Put (↑)
* Volatility (↑): Call Payoff (↑), Put Payoff (↑)
* Dividends (↑): Stock Price (↓), Call (↓), Put (↑)
* Interest Rate (↑): Call (↑), Put (↓)
* Time-Maturity (↑): European Options (↑/↓)

**Treasury Management**

**1: Foreign Exchange Market**

***Domestic in Terms of Foreign; Foreign in Terms of Domestic***

**2: Interest Parity Relationships**

***Interest Rate Arbitrage***

***Interest Rate No-Arbitrage***

***Absolute PPP***

***Relative PPP (w/ Inflation)***

**3: Exchange Exposure**

***Variance of a Two-Asset Folio***

***Variance of a Three-Asset Folio***

***Economic Exposure***

**4: Derivatives**

***Long/Sort Positions (Recall)***

***Forward Discount/Premium***

***Premium On Options***

**5: Interest Rate Risk**

* of a % is ‘Basis Point’
* Must Convert Period to Days

***Recall***

***Duration***

***Forward Rate Agreements***

***Interest Rate Option***

**Derivatives Extensive Summary**

**Lecture 1: Introduction to Derivatives**

* Derivative value depends on another asset
* E.g. (Derivatives): futures, forwards, swaps, options
* E.g. (Assets): equity, bonds, shares, interest rates
* Derivatives **transfer risk** in economy
* Derivative market larger than stock market (several \*global-GDP)

**1: How Are Derivatives Traded**

* Traded on standard der. exchanges
* Over-the-counter market (OTC) – traders work for banks, fund managers – corporate treasurers contact one-another directly

***1.1: Exchange-Traded Derivatives***

* Standardised contracts
* Limit credit risk
* Central clearing
* Regulated exchanges
* Liquid markets

***1.2: Over-the-Counter Markets***

* Tailor-made contracts
* Flexibility in negotiation
* Larger than exchange-traded market
* Telephone and computer network of dealers
* Higher **credit-risk**

**2: Forward Contracts**

* **Terms**: buy/sell at future time/price
* **Market**: Over-the-Counter
* Long-term maturity available
* Party agreeing to **buy** underlying asset: Long Position
* Party agreeing to **sell** underlying asset: Short Position
* *Legally binding agreement*

***2.1: Payoffs***

* From Long: From Short:

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**3: Futures Contracts**

* **Terms**: buy/sell at future time/price
* **Market**: Exchange-traded
* Available on commodities financial assets
* Standardised contracts
* Specification:
  + What can be delivered
  + Where it can be delivered
  + When it can be delivered
* Settled daily

**3: Options Contracts**

* **Call Option**: right to buy underlying asset on/by date for price
* **Put Option**: right to sell underlying asset on/by date for price
* **American Option**: exercised any time up-to expiration
* **European Option**: exercised only on expiration

***3.2: Definitions***

* **Premium**: price options bought/sold
* **Exercise/Strike Price**: price at which right to buy/sell underlying asset of call/put is set
* **Expirations/Maturity**: date in contract on/by option must be exercised
* **Writer**: supplier/seller of option at selling price (**premium**) (Short Position)
* **Holder/Investor**: party who’s acquired/bought option (Long Position)

***3.3: Options vs. Futures/Forwards***

* Futures/Forward is a **commitment** to buy/sell
* Options give the **right** to buy/sell
* Options traded on both **exchanges/OTC**
* Investors don’t pay premium to enter futures/forwards

**4: Types of Traders**

* **Hedgers**: use derivatives to reduce risk they face from potential movements
* **Speculators**: use derivatives to bet on future direction of a market variable
* **Arbitrageurs**: take offsetting positions in 2+ instruments to lock profit by taking adv. of price discrepancies

***4.1: Hedging***

* **Long Hedge**: one future date committed to **buying** assets, fix future price by taking Long Position in Futures on asset
  + “Hedge against possibility of price rise”
* **Short Hedge**: one future date committed to **selling** assets, fix selling price by taking Short Position in Futures on asset
  + “Hedge against possibility of price fall”
* In both, changes in value of asset can be offset by changes in value of position in futures

***4.2: Speculation***

* Betting on future changes in price of an asset using derivatives
* Bank of England “broken” example video

***4.3: Arbitrage***

* Riskless profit: simultaneous purchase and sale of asset to profit from difference in price
* Equivalent assets trade at different prices, they buy in cheaper markets and sell in more expensive one
* Rare and don’t last long
* Rational, well-formed markets: equivalent assets have same price (equilibrium: no arbitrage)

**Lecture 2: Futures Markets**

* Recall:
  + Long Position: Buyer
  + Short Position: Seller
  + Spot Price: Price at which can be sold immediately
  + Futures Price: Price at which can buy/sell in future
  + Basis: Difference between two
* Categories:
  + Agricultural commodities
  + Metals and minerals
  + Foreign currency
  + Financial futures

**1: Terminology**

***1.1: Convergence***

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* Towards expiration: futures converges on spot
* Point of expiration: futures and spot identical

***1.2: Terms***

* Open Interest: total contracts outstanding
  + Equal to no. of **long** or **short** positions
* Settlement Price: price right before final bell (each day)
  + For daily settlement processes
* Volume of Trading: no. trades – contracts traded – daily

***1.3: Patterns of Futures Prices***

* Normal Markets: Futures prices **pos. corr.** to time to maturity
* Inverted Markets: Futures prices **neg. corr.** to time to maturity

**2: Payoff from Futures (Gain/Loss)**

***2.1: Long Position & Short Position (Respectively)***

***2.2: Zero-Sum Game***

* All long positions offset by short position
* Therefore aggregate profit/loss w/ futures trading over all investors = 0
* Long: Short:

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**3: Margin Accounts**

* Margin is cash/marketable securities depo’d by investors w/ their broker (interest paying account)
* Determined by var. of price of underlying asset
* Balance of Margin Account is reflective of daily settlement
* Margins minimise possibility of loss through **default** on contract
* Initial Margin: amount which must be deposited at entry (usually 5%-15% of total value of contract)
* Marking to Market: account adjusted ea. trading day for gain/loss (daily settlement process)
* Maintenance Margin: if margin falls below critical value (75% of initial margin), investor receives margin call to restore level of initial margin
* Variation Margin: funds deposited in margin acc. following a margin call

**4: Clearing House**

* The trading partner of ea. side of futures contract
* Seller of contract for long position and buyer for short position
* Obligated to deliver commodity to long pos. and pay for delivery from short
* Zero-net-position
* Improves liquidity
* Reduces uncertainty and credit risk

***4.1: Margin CFs w/ Increase & Decrease in Futures Price (Respectively)***

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**5: Types of Orders**

* Market
* Limit
* Stop-Loss
* Stop-Limit
* Market-if-Touched
* Discretionary
* Time of Day
* Open
* Fill or Kill

**6: Forwards vs. Futures**

|  |  |
| --- | --- |
| * Forwards   + Private bet. Two parties   + Non-standard contract   + Usually one specified delivery date   + Settled at end of contract   + Delivery or final C settlement occurs   + Some credit risk | * Futures   + Exchange traded   + Standard contract   + Range of delivery dates   + Settled daily   + Contract usually closed out prior to maturity   + Virtually no credit risk |

**Lecture 3: Forward & Futures Prices**

**1: Investment vs. Consumption Assets**

* Investment Assets: held by significant numbers of people purely for investment purposes (e.g. stocks, bonds etc.)
* Consumption Assets: held primarily for consumption (e.g. oil, corn, etc.)

**2: Short Selling**

* Selling assets you don’t own
* Broker borrows securities from another client and sells in market
* In order to close your position: buy the securities to return them
* Div.’s and other benefits must be paid to owner
* Perhaps small fee for borrowing the assets

***2.1: Short Selling Example***

* Short 100 shares w/ price of £100
* Close short in 3 months w/ price of £90
* £ month div. of £3 per share
* Profit = (Selling Price – Buying Price)\*No. Shares – (Div.\*No. Shares)

= £700

***2.2: Notation***

***2.3: Assumptions***

* No transaction costs
* Same tax rates for all participants
* Borrow/lend at *r*
* Participants take adv. of arbitrage

**3: Pricing Futures: Spot-Futures Relation**

* Investor requires asset at time *T*

1. Buy asset today at *S0* and hold until *T* or…
2. Enter contract today to buy asset at *T* at *F0*

***3.1: Buy Asset Today***

* Cost of holding ‘til *T* is **interest lost** on alternative of holding C
* **Cost ‘Buy-and-Hold**’ = *S0* + Interest Lost
* Cost of holding asset is reduced by any **income**, e.g. div.’s/shares
* **Cost of ‘Buy-and-Hold’** = *S0* + Interest Lost – Income Received

***3.2: Is There Arbitrage?***

* Suppose:
  + *S0* of non-div-paying stock is £40
  + *F0* (futures price) is unknown
  + 3-month (forward price) is £43 or £39 (Arbitrage)
  + 3-month UK *i* is 5% p.a.
* Forward Price:
  + **Standard**:
  + because cost of financing the spot purchase of asset during life of project
  + **Arbitrage**:
    - Short one share for £40
    - Invest proceeds in short sale for *F0*
    - Take **long-forward** to buy share at £39 in 3 months to close short
    - Take **Riskless** **Profit** = £40.50 - £39 = £1.50
* Price Equilibrium:
  + : arbitrageurs can buy asset and enter **short-forward** contract on asset for **Riskless Profit** =
  + : arbitrageurs can short sale asset and enter **long-forward** on asset for **Riskless Profit** =

**4: Short-Selling Not Possible**

* See Slide 13 and on

**Lecture 4: Hedging Using Futures**

* …

**Lecture 5: Interest Rate Swaps (IRSs)**

* A swap is an over-the-counter agreement
  + Two parties
  + Exchange CFs
  + Specified future times – according to rules

**1: Plain Interest Swaps**

* Agrees: to make fixed *i* payments on **notional** **principal** – given no. years
* In return: receives *i* at a floating rate on same **notional principal** – same period
  + Notional principal: used only to calculate of *i* payments – not exchanged
* **Example**:
  + *MSFT agrees to receive 6-month LIBOR and pay Intel fixed rate of 5% every 6 months for 3 years – on notional principal of £100m*
    - Microsoft: Long Position in floating-rate bond & Short Position in fixed-rate bond
    - Intel: Long Position in fixed-rate bond & Short Position in floating-rate bond

**2: Uses of IRSs**

* Covert Liability from:
  + Fixed-rate to floating-rate
  + Floating-rate to fixed-rate
* Convert Investment (Asset) from:
  + Fixed-rate to floating-rate
  + Floating-rate to fixed-rate

***2.1: Example: Converting Liability***

* MSFT borrow £100m at *i* of LIBOR + 0.1%
* Receives LIBOR on **notational principal** of £100m from Intel
* Pays 5% on **same** **principal**
* –
* MSFT NetLiability is **fixed-rate** *i* payment of:
  + (LIBOR + 0.1% - LIBOR + 5%) = 5.1% on £100m

**3: Market Makers (Swap Dealers)**

* Unlikely two companies will need to oppose positions in same swap at same time
* Large financial institutions act as Market Makers for swaps
* Market Makers prepared to enter swap w/o having offsetting/counterparty swap
* Carefully quantify and hedge risks they take

***3.1: Example: Converting Liability w/ Financial Institution***

* MSFT borrow £100m at *i* of LIBOR + 0.1%
* Receives LIBOR on **notational principal** of £100m from Intel
* Pays 5.015% via Financial Institution to Intel
* Financial Institution pays Intel 4.985% on £100m
* –
* MSFT NetLiability is **fixed-rate** *i* payment of:
  + (LIBOR) = 0.1% - LIBOR + 5.015% = 5.115% on £100m
* Financial Institution makes profit of:
  + 0.03% (= 5.0155 – 4.985%)

***3.2: Example: Converting Assets***

* MSFT bought bonds of £100m which provide 4.7% p.a. for 3 years
* Receives LIBOR on **notional principal** of £100m from Intel
* Pays Intel 5% on **same principal**
* –
* MSFT Net Interest Rate Inflow on £100m:
  + 4.7% + LIBOR – 5% = LIBOR – 0.3%

***3.3: Example: Converting Assets w/ Financial Institution***

* MSFT bought bonds of £100m which provide 4.7% p.a. for 3 years
* Receives LIBOR on **notional principal** of £100m from Intel
* Pays Intel 5.015% via Financial Institution to Intel
* Financial Institution pays Intel 4.985% on £100m
* –
* MSFT Net Interest Rate Inflow on £100m:
  + 4.7% + LIBOR – 5.015% = LIBOR – 0.315%
* Financial Institution makes profit of:
  + 0.03% = 5.015% – 4.985%

**4: Total Gains**

* Total Gain form IRS = Difference in Fixed Rates – Difference in Floating Rates
  + Slide 14 for More

**5: Credit Risk**

* Suppose: *i* increases right after interest rate agreement begins
* **Floating rate** payer suffers **loss** and backs out
* Loss for **fixed-rate** payer limited to difference between fixed and floating rates
* Default of **floating-rate** payer relieves **fixed-rate** payer from obligation too

**6: Forward Rate Agreement (FRA) Valuation**

* Value of FRA is PV of difference between *i* paid at RFL and *i* which would be paid at RFRA

**7: Interest Rate Swap Valuation**

* Initially: worth close to 0
* As time goes on: equal to difference between **fixed-rate** bond and **floating-rate**
* **Floating-rate** payer:
* **Fixed-rate** payer:
* Alternatively: valued as portfolio of Forward Rate Agreements

***7.1: Example: w/ Microsoft***

* LIBOR rate 5%
* Risk-free rate 4%

**8: Overnight Index Swaps (OIS)**

* **Fixed-rate** for a period is exchanged for geometric avg. of Overnight rates
* Allows overnight borrowing/lending swapped at **fixed-rate**
* **Fixed-rate** in OIS referred to as “Overnight Swap Rate”
* Bears risk that counterparty (another bank) will default
* To compensate: LIBOR > OIS
* Example: Slide 29

**9: Currency Swaps**

* Exchange principal and *i* payments in one currency for principal and *i* payments in another currency
  + Convert liability in one currency to a liability in another
  + Convert investment in one currency to investment in another
  + Quick, cheap, anonymous method of restructuring balance sheet

**Lecture 6: Options Markets**

**1: Options vs. Forwards/Futures**

* Right to do something – does not have to exercise
* As opposed to commitment (forwards/futures)
* Forwards/futures do not require time 0 payments
* Options require time 0 payments
* Positions:
  + **Long Call**: buy right to buy *(bullish)*
  + **Long** **Put**: buy right to sell *(bearish)*
  + **Short** **Call**: sell right to buy *(bearish)*
  + **Short** **Put**: sell right to sell *(bullish)*

**2: Long Call Option (1:4)**

A close up of a clock

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* C0 = Call Option Price (e.g. £5)
* K = Strike Price (e.g. £100)
* “” = Breakeven = C0 + K
* Exercise only if share price at expiry is greater than strike price
  + If ST > K: Exercise
  + If ST < K: Don’t Exercise
* **Example**: C = 8; K = 100
  + Profit/Loss = max{ST – K – C0}, – C0
  + Assume ST = 128
    - P/L = max{128 – 100 – 8}, – 8
    - P/L = max{20, -8} = 20 so Exercise
  + Assume ST = 90
    - P/L = max{90 – 100 – 8}, – 8
    - P/L = max{18, -8} = -8 so Don’t Exercise
  + Assume ST = 105
    - P/L = max{105 – 100 – 8}, – 8
    - P/L = max{-3, -8} = -3 so Exercise to Minimise Loss

**3: Long Put Option (2:4)**

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* P0 = Put Option Price (e.g. £7)
* K = Strike Price (e.g. £70)
* Breakeven (Shown). = K – P0
* Exercise if profit from selling asset through put is greater than market price
  + If ST < K: Exercise
  + If ST > K: Don’t Exercise
* **Example**: P = 8; K = 100
  + Profit/Loss = max{K – ST – P0}, – P0
  + Assume ST = 128
    - P/L = max{100 – 128 – 8}, – 8
    - P/L = max{-36, -8} = -8 so Don’t Exercise
  + Assume ST = 90
    - P/L = max{100 – 90 – 8}, – 8
    - P/L = max{2, -8} = 2 so Exercise
  + Assume ST = 98
    - P/L = max{100 – 98 – 8}, – 8
    - P/L = max{-6, -8} = -6 so Exercise to Minimise Loss

**4: Short Call (3:4)**

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* C0 = £5
* K = £100

**5: Short Put (4:4)**

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* P0 = £7
* K = £70

**6: Call & Put Profitability**

* Call Profitable: ST > (C0 + K)
* Put Profitable: K > (P0 + ST)
* Call Option Zero-Sum Game: Put Options Zero-Sum Game:

A close up of a map

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* **A picture containing object, antenna

  Description automatically generatedSummary**:

**7: Value At Expiration**

* Call:
  + ST – K **if** ST > K
  + 0 **if** ST K
* Put:
  + K – ST **if** ST < K
  + 0 **if** ST K

**8: Moneyness**

* In-the-Money Option (profitable when):
  + Call: S > K
  + Put: S < K
* At-the-Money Option
  + S = K
* Out-of-the-Money Option

**9: Intrinsic & Time Value**

* Option Price (Premium) = Intrinsic Val. + Time Val.
* Intrinsic Value maximum of 0, option value would have it if instantly exercised
  + Call: max{S – K, 0}
  + Put: max{K – S, 0}
* Time Value difference between current option price and intrinsic value

**10: Market Makers**

* Quotes **bid** (price prepared to buy) and **ask** (price prepared to sell) prices
* **Bid-Offer Spread**: difference between **bid** and **ask**

**11: Option-Like Securities**

* Callable Bonds
* Convertible Bonds (and other securities)

**Lecture 7: Properties of Stock Options**

**1: Notation & Introduction**

|  |  |
| --- | --- |
|  |  |

***1.1: Factors Affecting Option Prices***

* Current stock price,
* Strike price, *K*
* Time to expiration, *T*
* Volatility of stock price,
* Risk-free rate, *r*
* Dividends expected, *D*

***1.2: Interaction Matrix***

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Variable** |  |  |  |  |
|  | + | – | + | – |
|  | – | + | – | + |
|  | ? | ? | + | + |
|  | + | + | + | + |
|  | + | – | + | – |
|  | – | + | – | + |

***1.3: American vs. European Options***

* American worth at least as much as corresponding European

***1.4: Upper Bounds***

* **Call** can never be worth more than **underlying stock**
* **Put** can never be worth more than **PV of K**

**2: Call Values at *T***

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Investment** | **Value at *T***  (In the Money)  *E.g. ST = £190 > K* | **Value at *T***  (Out of the Money)  *E.g. ST = £130 < K* |
| **Portfolio A** | Invest in *S0* = £140  **Total = *S0*** | *ST* = £190  **= £190** | *ST* = £130  **= £130** |
| **Portfolio B** | Call (*C0* = £20)  +  PV of K (*K* = £150)  **Total = C*0* + PV(K)** | *Exercise Call*  *K – K + ST*  **= £190** | *Do Not Exercise*  *K*  **= £150** |

***2.1: Lower Bound***

* If folio B offers > folio A, worth more at t=0:
  + *C0 + PV(K) S0*
  + *C0 S0* – *PV(K)*
  + *C0 S0* – *Ke-rT*

**3: Put Values at *T***

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Investment** | **Value at *T***  (In the Money)  *E.g. ST = £130 < K* | **Value at *T***  (Out of the Money) *E.g. ST = £190 > K* |
| **Portfolio C** | Put (*P0* = £30)  Call (*S0* = £140)  **Total = *S0*** | *Exercise Put*  *K*  **= £150** | *Do Not Exercise*  *ST*  **= £190** |
| **Portfolio D** | PV of K (*K* = £150)  **Total = PV(K)** | *K* =150  **= £190** | *K*  **= £150** |

***3.1: Lower Bound***

* *P0 + S0 PV(K)*
* *P0 PV(K) – S0*
* *P0 Ke-rT – S0*

**4: Put-Call Parity**

* *“Since folio B gives equal payoffs at T as folio C, must have same payoffs at 0”*
  + *C0* + *PV(K)* = *P0 + S0*
  + Assume **continuous** **compounding**:
    - *C0* + *Ke-rT* = *P0 + S0*

**5: Early Exercise**

* Chance w/ American option
* Exception where it is a non-div. paying stock (don’t exercise early)
* Should **never** be exercised early if investor plans to hold stock for life of option
* **Do not** early exercise if:
  + No income is sacrificed
  + You delay paying the strike price
  + Holding the call provides insurance against *S0* falling below *K*

**6: Graphical Representation**

* A screenshot of a cell phone

  Description automatically generatedEuropean Call: European Put:

A screenshot of a cell phone

Description automatically generated

* American Put:

A close up of a device

Description automatically generated

**Lecture 8: Trading Strategies w/ Stock Options**

* Strategies
  + Option plus underlying asset
  + Two or more options – same type (**Spread**)
  + Two or more options – different type (**Combination**)

**1: Covered Call**

* Write call, invest in underlying asset (hold share, write call)
* Controls risk exposure – call writers
* Earn premium on holdings by giving up potential high returns – share holders

A close up of a map

Description automatically generated

* Profit /Loss
  + Exceeds profit on simple investment for all prices up to exercise (plus call value)
  + Combination – lower profit than would be seen holding normal share for prices exceeding exercise (plus call value)
* Note: short stack and long call payoffs (Slide 6)

**2: Protective Put**

* Insurance against **share price fall** 
  + P0 = Put Premium
  + X – Exercise Price
  + S0 = Share Price at 0
* If S falls below S0, share can be sold through Put Option to realise (X = S0)
* Investor benefits: any increase in S above S0, guaranteed min. payoff (S0 – P0)

**A close up of a map

Description automatically generated**

* Note: short stack and short put payoffs (Slide 9)

**3: Spreads**

* Calls or Puts
  + Bull Spreads
  + Bear Spreads
  + Box Spreads
  + Butterfly Spreads

***3.1: Bull Spread***

* Limits up/downside risk
* Three types:
  + Both options are **out of the money** initially
  + One is **in the money** and one **out the money** initially
  + Both options **in the money** initially
* *“Investor expects S* ***Increase****”*

**Lecture 9: Option Pricing**

* **Binomial Tree**: representation of different Option paths as probabilities
  + Assumes – stock prices follow ‘random walk’
  + Assumes – no arbitrage opportunity
  + As steps get smaller, model converges to Black & Scholes Merton

**1: One-Step The Binomial Tree**

***1.1: Simple Tree***

* Call Option: *“Value of European Call w/ Exercise of £21?”*

A close up of a logo

Description automatically generated

* Riskless Folio:
  + Long Position – Change in Shares
  + Short Position – 1 Call Option

A close up of a logo

Description automatically generated

* + Riskless when:
  + **Example**:
    - Stock Price = £22;
    - Stock Price = £18;
    1. *// Riskless Folio – Earns 12% //*
    2. *// Value of Call Option – Stock today recall = £20 //*

***1.2: Notation***

* ***A close up of a logo

  Description automatically generated***Where: {u > 1; 0 < d < 1}
* **Generalise**

***1.3: Generalised Form***

***A picture containing antenna

Description automatically generated***

* Riskless When:

*“Ratio of Option Price change to Stock Price change”*

* + Value of Folio (at *T*):
  + Value of Folio (at *T*, When Riskless):
  + Value of Folio (at 0):
  + **Value of Call (at 0)**:
    - *// Substitute //*

A picture containing object

Description automatically generated

**2: Two-Step Binomial Tree**

A picture containing sky, map

Description automatically generated

**3: Girsanov’s Theorem**

* is the **same** through real-world and risk-neutral-worlds
* Can therefore measure in real-world and use it in risk-neutral

***3.1: Choosing u & d***

**4: Probability of An Upwards Move**

* + : Non-Div. Stock
  + : Div. Stock
  + : Foreign Risk-Free Rate
  + : Futures Contract

**Lecture 10: Black & Scholes Model**

* Function of **underlying** **asset** and **time**
* Two parameters: **risk-free rate** and **asset volatility**
* *“The history of the variable is irrelevant”*
  + Short-selling permitted
  + No transaction costs
  + No risk arbitrage
  + Rf and volatility are constant
  + No dividends from underlying assets

**1: Notation & Definitions**

***1.1: Normal Probability***

* *N(x)* is probability that a **normally distributed** variable w/ mean of 0 and std.dev of 1, is less than *x*

A picture containing drawing, game

Description automatically generated

**2: Black & Scholes: Call Option**

***2.1: Call Option***

* : risk-adjusted probability that Call Option will expire **in-the-money**
* : percentage amount by which Call Option is **in/out-of-the-money** 
  + **Example**: *S* = 160, *X* = 100; Option is 60% **in-the-money** at 0.47
  + **Example**: *S* = 90; *X* = 100; Option is 10% **out-of-the-money** at -0.105
* Hence, when *N(d)* is close to 1, there’s near a 100% chance the Call will be exercised
  + If *N(d1)* and *N(d2)* are 1, Call Value will be approx. its intrinsic vlue
* When there’s a yield on the underlying asset:

***2.2: Put Option***

* The **Put-Call Parity**

***2.3: Properties of Model***

* As *S0* gets large: *C0* tends towards (*S0* – *Xe-rT*); *P0* tends to 0
* As *S0* gets small: *C0* tends towards 0; *P0* tends to (*Xe-rT* – *S0*)
* As tends to 0: *C0* tends towards max{*S0* – *Xe-rT*, 0}

*C0* tends towards max{*Xe-rT* – *S0*, 0}

* If including **div.**, they should be the ex-div. rather than div.
  + Should be the E(reduction in *S*)
  + Short-Life Options: estimate div. during option life w/ decent accuracy
  + Long-Life Options: estimate div. w/ uncertainty – option pricing difficult

**3: Volatility**

* Estimated from historical data – suing daily R’s over several months
* Volatility greater when market open
* Thus, usually measured in **trading days** rather than calendar days (252 days/yr.)
  + **Example**: April 1st 🡪 April 30th: 22 t-days: T = (22/252 = 0.0873yr.)

***3.1: Implied Volatility***

* for which Black-Scholes price = Market Price
* *Forward-looking (where normal volatility is backward)*
  + *High corr. w/ financial crisis, known as “investor fear gauge”*
* One-to-one corr. between prices and **implied volatility**
* If actual : Option good to buy
* If actual : Option price lower than observed

***3.2: Estimated Volatility Incorrect?***

* Higher : increases option prices; Lower : decreases option prices
* May be B-S biases
* Mkt. prices may be incorrect – e.g. market inefficiency
  + If market **out** of equilibrium **arbitrage** exists

**4: Option Payoff at Expiration**

* *X* > *ST*:
  + Put exercised to receive (***X* – *ST***)
  + Call not exercised
* *X* < *ST*:
  + Put not exercised
  + Call exercised to receive –(*ST* – *X*) = (***X* – *ST***)

***4.1: Overall Position at Expiration***

* Share Value: (+) *ST*
* FV of Div.: (+) *FV(D)*
* Option Payoff: (+) (*X* – *ST*)
* Repay Loans: (–) (*FV(D) – X*)
* **Net: 0**

**5: Arbitrage & Biases in the Model**

* Mispricing should be temporary – market should return to equilibrium
* Arbitrageurs don’t need to hold until maturity
  + Buy puts at low then when equilibrium is reached, sell at high
* Based on the European option – doesn’t account for early exercise however, American Calls on non-div. never early-exercise so basically same as European
* Early Put more common – div. or not. If sufficiently **in-the-money**, w/ long (*T-t*), may be beneficial to early-exercise